Homework Feedback 12

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Weiwei Xu

**P. 271 #5** Given the initial-value problem:

with exact solution .

a. Use Taylor's method of order two with h = 0.1 to approximate the solution, and compare it with the actual values of y.

b. Use the answers generated in part (a) and linear interpolation to approximate y at the

following values, and compare them to the actual values of y at 1.04, 1.55,1.97

Answer: (a) Take Taylor method of order 2 as an example. First, let us derive :

Therefore, the order 2 Taylor method is the following:

We get the values of y with h=0.1:

|  |  |  |
| --- | --- | --- |
| t | y(t) | error |
| 1 | 0 | 0 |
| 1.1 | 0.339785 | 0.006135 |
| 1.2 | 0.852143 | 0.014499 |
| 1.3 | 1.581770 | 0.025446 |

(b) perform linear interpolation to get the y values

**P. 280 #5** Use the Modified Euler method to approximate the solutions to each of the following initial value problems, and compare the results to the actual values.

Answer: The modified Euler method is the following:

Take (a) as an example:

|  |  |  |
| --- | --- | --- |
| t | y(t) | error |
| 0 | 0 | 0 |
| 0.5 | 0.560221 | 0.276594 |
| 1.0 | 5.301490 | 2.082391 |

**P. 281 #10** Repeat Exercise 1 using the Runge-Kutta method of order four.

|  |  |  |
| --- | --- | --- |
| t | y(t) | error |
| 0 | 0 | 0 |
| 0.5 | 0.296997 | 0.01338 |
| 1.0 | 3.314312 | 0.095213 |

**P. 281 #13** Show that the Midpoint method, the Modified Euler method, and Heun's method give the same approximations to the initial-value problem:

for any choice of h. Why is this true?

Answer: We can derive that all three methods lead to the same formula:

This is because that is a linear function.